Announcements

1) Advising Sessions
3-4 CB 2047 (Math Library)
Pizza - M + T

2) Urban Science Internships
Ask me

3) Math Club talk M 4:10-5
Michelle Intermont
"Change-Ringing"
Mathematical Music

Fun from Cantor

The Cantor Set.

This is a subset of $[0,1]$.

How it is constructed:

Take $[0,1]$, throw away $(1/3, 2/3)$ (middle third)

Stage 1

Now take

$$[0, 1/3] \cup [2/3, 1],$$

throw away "middle thirds" — the intervals $(1/9, 2/9)$ and $(7/9, 8/9)$. We get

Left with $[0, 1/9] \cup [2/9, 1/3] \cup$

$[2/3, 7/9] \cup [8/9, 1]$

Stage 2.

Iterate the procedure, take out "middle thirds" over and over again. What's left after all the iterations?

Not the empty set!

All endpoints of intervals whose "middle thirds" have been removed remain.

There are countably many endpoints.

However, if we take

Cantor Set = all the points remaining (not just endpoints),

you can show that this set is uncountable!

However, the (infinite) sum of all the lengths of intervals deleted is equal to one!

The Cantor set is therefore "in between" dimensions — length zero, but uncountable.

There is a notion of dimension that ascribes $\frac{\ln(2)}{\ln(3)}$ to the dimension of the Cantor set!

Chapter 3

"Topology of Metric spaces"

Recall: X is a metric space

if there is a function

$d: X \times X \to [0, \infty)$

satisfying

1) $d(x,y) = 0$ if and only if $x = y$
2) $d(x,y) = d(y,x)$
3) $d(x,z) \leq d(x,y) + d(y,z)$

$\forall \; x, y, z \in X$

Definition: (ball) If $x \in X$, where X is a metric space, we define the ball of radius ε for $\varepsilon > 0$ as

$$B(x,\varepsilon) = \underbrace{\{y \in X \mid d(x,y) < \varepsilon\}}_{\text{Ball of radius } \varepsilon}$$

In $\mathbb{R}$ $\quad d(x,y) = |x-y|$,

so

$$B(x,\varepsilon) = \{y \in \mathbb{R} \mid |x-y| < \varepsilon\}$$

Note

$|x-y| < \varepsilon$ means

$-\varepsilon < x-y < \varepsilon$ which

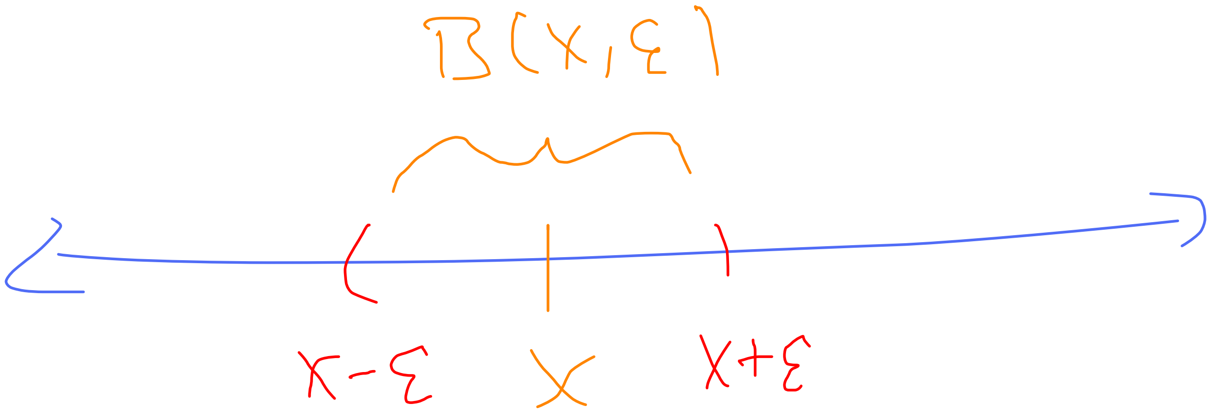
is equivalent to

$-\varepsilon < y-x < \varepsilon$. Adding x,

$$\boxed{x-\varepsilon < y < x+\varepsilon}$$

So

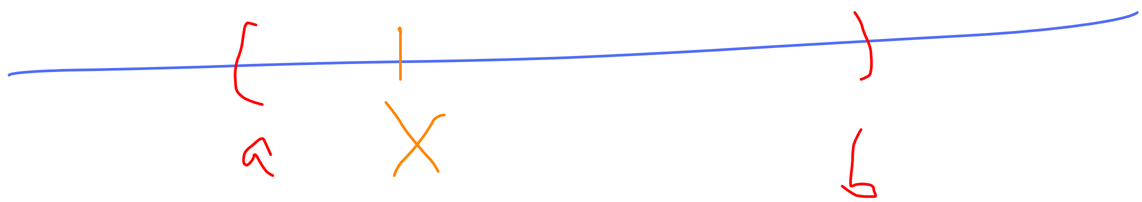
$B(x,\varepsilon) =$ open interval of length 2ε centered at x



Definition: (open set)

A subset S of a metric space X is said to be **open** if $\forall x \in S$, $\exists \varepsilon > 0$ (dependent on x!) with $B(x, \varepsilon) \subseteq S$.

Example 2: ($\mathbb{R}$) Any open interval (a,b) is an open set: if $x \in (a,b)$



let $\varepsilon = \min\left\{ \frac{x-a}{2}, \frac{b-x}{2} \right\}$

Then if $y \in B(x,\varepsilon)$,

$y \in (a,b)$ (check!).